THEORY OF ELECTRODIFFUSION MEASUREMENTS OF FRICTION IN MICRODISPERSED LIQUIDS. DIFFUSION LAYER APPROXIMATION

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1. Introduction. The problem of mass transfer for a friction sensor is solved in the concentration boundary layer approximation for a fairly broad class of velocity profiles. A discussion is presented of the question of determining boundary velocity profiles from measurements of limiting diffusion flux.

In the electrodiffusional (ED) measurement of friction in Newtonian fluids [1], it makes no difference how the results are interpreted: in the form of shear stresses at the wall τ or in the form of the corresponding shear velocities γ. The matter is quite different, however, in the case of ED measurements made in clayey suspensions, polymer solutions, and other microscopically disperse liquids [2], since anomalous boundary effects are often seen in these instances. From the standpoint of convective diffusion, this difference is expressed in the fact that the velocity profiles in the immediate vicinity of the wall cannot be assumed to be linear. When analyzing raw data on limiting diffusion currents in cases such as these, it is necessary to represent the total current of the sensor I as a certain response functional whose arguments are the profile of velocity $v_x = u(z)$ inside the diffusion layer of the sensor and the length of the sensor along the liquid flow h.

In light of this, it turns out that simultaneous measurement of the limiting diffusion currents for a group of sensors of different lengths is the most effective method of employing ED-techniques to study the fine structure of the boundary velocity profile in microdisperse liquids [3, 4]. In the present study, we assume that ED-measurements of this type have been made for the case in which we use an ED-depolarizer with a known and constant diffusion coefficient D.

The function $\delta = \delta(h)$ is known for a certain range of sensor lengths h, where mean diffusion thickness δ is determined from raw test data: $\delta = nFc_0DA/I$ (nF is the molar charge of the electrochemical reaction, c_0 is the initial concentration of the depolarizer, and A is the area of the sensor). We need to find the operator $\delta(h) \mapsto u(z)$ that transforms the data on $\delta = \delta(h)$ into information on the boundary profile of velocity $u = u(z)$ (z is the distance to the surface of the sensor).

2. Problem of a Friction Sensor. It is not difficult to formulate a problem whose inverse problem is construction of the operator $\delta(h) \mapsto u(z)$. It consists of the solution of the parabolic equation

$$u(z)\,\partial_x c = D\partial_{zz}^2 c \tag{2.1}$$

with boundary conditions for the limiting current on the surface of the sensor:

$$c|_{z=0} = 0 \quad \text{for} \quad (x,\ y) \in A; \tag{2.2}$$

the condition of inertia of the walls surrounding the sensor:

$$\partial_z c|_{z=0} = 0 \quad \text{for} \quad (x,\ y) \notin A, \tag{2.3}$$

and the condition expressing the fact that the solution does not become depleted of depolarizer:

$$c \to c_0 \quad \text{for} \quad z \to \infty. \tag{2.4}$$

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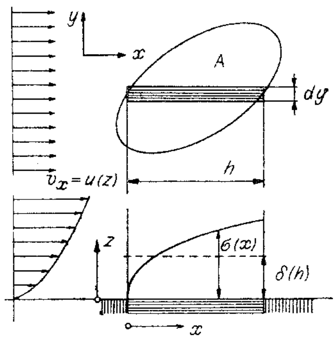

Fig. 1.

The solution of the problem will be the three-dimensional concentration field $c = c(z;\ x,\ y)$ and the corresponding total current $I = nF \iint_A j\ (x,\ y)dxdy$, where j is the local density of the diffusion current flowing to the surface of the sensor $j = -D\partial_z c|_{z=0}$.

The local three-dimensional problem can be considered two-dimensional in the calculation of local current densities, since the transverse coordinate is immaterial in the parabolic boundary-value problem being examined (in which we ignore the effect of longitudinal and lateral diffusion). The formulation of the two-dimensional problem is evident from Fig. 1. For an elementary strip $h \times dy$, we can shift the longitudinal coordinate x so that its origin coincides on the given boundary streamline (y = const) with the leading edge of the sensor. The corresponding two-dimensional problem consists of differential equation (2.1) with boundary conditions (2.2), (2.4) on the interval $0 < x < h$ and initial condition $c \to c_0$ for $x \to 0$ at $z > 0$ (replacing conditions (2.3) and (2.4) for $x < 0$).

We can use the resulting concentration field to determine the local diffusion thickness σ or the local density of the diffusion current flowing to the wall j: $j(x) = -D\partial_z c|_{z=0} = Dc_0/\sigma(x)$. We can then determine the mean density J(h) or mean diffusion thickness $\delta(h)$ on the elementary strip:

$$J(h) = h^{-1}\int_0^h j(x)\,dx = Dc_0 h^{-1}\int_0^h \sigma^{-1}(x)\,dx = Dc_0/\delta(h).$$

For a strip-type friction sensor, macroscopic mean values of J(h) and δ(h) can be determined directly from the raw data.

3. Similar Solutions, Locally Similar Approximations. Substitution of the new variables $c(z,\ x) = c_0 F(\xi,\ \sigma)$, $\xi = z/\sigma(x)$ reduces the problem to the form

$$F'' + \frac{d\sigma}{dDx}\sigma u(\sigma\xi)\,\xi F' = \frac{d\sigma}{dDx}\sigma^2 u(\sigma\xi)\,\partial_\sigma F; \tag{3.1}$$

$$F(0,\ \sigma) = 0,\ F(\infty,\ \sigma) = 1,\ F'(0,\ \sigma) = 1, \tag{3.2}$$

where F′ and F″ are partial derivatives with respect to ξ. The problem is ill-conditioned, since we lack initial conditions for $\sigma \to 0$, i.e., $x \to 0$. In such cases, it is customary in boundary layer theory to add the condition of local similitude at $x \to 0$:

$$\partial_\sigma F(\xi,\ \sigma) \to 0 \ \text{ for } \ \sigma \to 0. \tag{3.3}$$

In general, a locally similar approximation consists of ignoring the term in the right side of Eq. (3.1) i.e., of assuming that $\partial_\sigma F(\xi,\ \sigma) = 0$. With this simplification, we can obtain an explicit expression for the concentration field

$$F'(\xi, \sigma) = \exp\left[-\frac{d\sigma}{dDx}\sigma\int_0^\xi u(\sigma t)\,t\,dt\right],\quad F(\xi, \sigma) = \int_0^\xi F'(s, \sigma)\,ds$$

with an implicit integrodifferential equation for the sought diffusion thickness:

$$F(\infty, \sigma) = 1 = \int_0^\infty \exp\left[-\frac{d\sigma}{dDx}\sigma\int_0^s u(\sigma t)\,t\,dt\right]ds \tag{3.4a}$$

or

$$\sigma = \int_0^\infty \exp\left[-\frac{1}{xD}\frac{d\ln\sigma}{d\ln x}\int_0^z u(z_1)\,z_1\,dz_1\right]dz. \tag{3.4b}$$

In the special case, examined in [3, 4], where the velocity profiles are represented by a power function

$$u(z) = Bz^p, \tag{3.5}$$

we can use (3.4b) to construct the function $\sigma = \sigma(x)$ in implicit form

$$xD = B(2+p)^{-2}\left[\frac{\sigma}{\Gamma}\left(\frac{3+p}{2+p}\right)\right]^{(2+p)} \tag{3.6}$$

and we can obtain the corresponding representation for the mean diffusion thickness

$$\delta(h) = \frac{1+p}{2+p}\sigma(h) = \frac{1+p}{2+p}\Gamma\left(\frac{3+p}{2+p}\right)\left[\frac{(2+p)^2hD}{B}\right]^{1/(2+p)}. \tag{3.7}$$

4. Exact Non-Similar Solutions, Principle of Superposition. The method of local similitude (3.4) can be used to find the exact solution only for exponential velocity profiles. In other cases, it is best to solve parabolic problem (3.1)-(3.3) with an unknown eigenfunction $\sigma = \sigma(x)$. Calculations of this nature were published in [5] for the general linear profile $u(z) = U + \gamma z$. The authors also obtained results for the class of profiles $u(z) = \Sigma B_p z^p$ for three terms of the sum: $0 \le p_1 < p_2 < p_3 \le 2$, $B_1 > 0$, $B_2 \ne 0$. In all cases, the accuracy of the functions $\sigma = \sigma(x)$ was estimated as 5-6 significant digits for σ in the determination of x. It turned out that the results (to within 3-4 significant digits for σ) could be reduced to surprisingly simple generalizing conclusions which can be represented by the following empirical superposition principle.

For the class of velocity profiles $u = u(z)$, which are nondecreasing functions, we represent the relation $x = x(\sigma)$ in the form of a linear velocity functional with the parameter σ:

$$Dx = \Phi[u(z); \sigma],\ \Phi[B_1u_1(z) + B_2u_2(z); \sigma] = B_1\Phi[u_1(z); \sigma] + B_2\Phi[u_2(z); \sigma].$$

An example of the usefulness of this conclusion is that, for the class of exponential profiles $u_p(z) = B_p z^p$, we have an explicit representation of functional Φ in the form of Eq. (3.6).

5. Localization of the Functional for Mean Diffusion Thicknesses. We write Eq. (3.7) in the form $B(\alpha\delta)^p = u(\alpha\delta) = \psi(p, \alpha)hD/\delta^2$, where $\psi(p, \alpha) = \alpha^p(2+p)^2\left[\frac{1+p}{2+p}\Gamma\left(\frac{3+p}{2+p}\right)\right]^{2+p}$.

With the particular choice of $\alpha = 0.414$, the value of ψ satisfies the condition $\psi(0, \alpha) = \psi(1, \alpha) = 0.785$, and for $0 \le p \le 1$ we have a nearly constant value $0.78 < \psi < 0.83$. As a result, the following empirical formula has been proven valid for the class of exponential velocity profiles at $0 \le p \le 1$

$$u(z)|_{z=0.4\delta} = 0.8hD/\delta^2. \tag{5.1}$$

In accordance with this formula, the mean diffusion current of the friction sensor depends on the local value of velocity at the distance $z = 0.4\delta$ to the wall.

Within the above-indicated error (±5%), Eq. (5.1) can be used for a more general class of velocity profiles. In particular, the following representation is obtained from the superposition principle for the linear velocity profile $u(z) = U + \gamma z$:

$$Dh/\sigma^2 = \frac{U}{4\Gamma^2(3/2)}(1+b),\quad \delta = \frac{1}{2}\sigma(1+b)/(1+0.75b),$$

where $b = 0.49\gamma\sigma/U$ and $\sigma = \sigma(h)$, $\delta = \delta(h)$. The ratio $u(0.4\delta)\delta^2/(0.8hD) = (1 + 0.56b)(1 + b)/(1 + 0.75b)^2$ deviates from unity by no more than 2%.

6. Conclusions. The superposition principle, employed here for a strip-type sensor, remains in force for other types of sensors with the same numerical coefficients. In particular, Eq. (5.1) is valid for a circular sensor of radius R if we make the substitution $h = 1.64R$. Equation (5.1) can be regarded as an empirical representation of the inverse operator $\delta(h) \mapsto u(z)$ in the problem of the friction sensor. Taking this into account, we intepret the power representation of velocity profile (3.5) as follows:

$$p = \frac{d \ln u}{d \ln z}\Big|_{z=0,4\delta} = \frac{d \ln h}{d \ln \delta} - 2; \tag{6.1}$$

$$B = \left(u(z)/z^p\right)\big|_{z=0,4\delta} = \frac{0,8hD}{0,41^p \delta^{2+p}}. \tag{6.2}$$

Equations (5.1), (6.1), and (6.2) can be used directly to analyze electrodiffusion data for microdisperse fluids in which wall slip effects are seen.

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